Dynamics of mobile impurity in one-dimensional quantum gas

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Several theoretical results on the dynamics of mobile impurity in one-dimensional quantum gas are reported. Quantum gas is modeled by noninteracting fermions. Due to boson-fermion correspondence valid in 1D this is equivalent to the Tonks-Girardeau gas of impenetrable bosons [1]. The interaction between the impurity and the host fermions is pointlike and repulsive: $\hat{V} = g \int dx \hat{\rho}_h(x)\hat{\rho}_i(x)$, where $\hat{\rho}_h(x)$ and $\hat{\rho}_i(x)$ are density operators of the host fermions and the impurity, respectively. The mass of the host particles is taken to be 1 while the mass of the impurity is m_i. At $t = 0$ the fermions are at equilibrium at zero temperature while the impurity is prepared in a state with a well-defined momentum p_0. We are interested in the average impurity momentum as a function of time and initial momentum, $p(t; p_0)$, and especially in the asymptotic impurity momentum $p_\infty(p_0) \equiv p(\infty; p_0)$.

In the $m_i = 1$ case the model is solvable via Bethe ansatz [2, 3]. Recently this case was studied in Ref. [4] in the strong coupling regime, $g > 1$. In particular, it was argued that the asymptotic impurity momentum is nonzero. This conclusion was based on a computer simulation which is unavoidable while handling Bethe solution.

We concentrate on the weak coupling case which permits perturbative study. The main results we report are as follows:

- An exact lower bound on $p_\infty(p_0)$ for $|p_0| < r_0 \equiv \min\{1, m_i\} \cdot k_{\mathrm{F}}$ is obtained. The existence of this bound rigorously proves that the impurity momentum does not relax to zero, at least for $|p_0| < r_0$.

- It is shown that in the zeroth order in g^2 the asymptotic impurity momentum as a function of the initial momentum satisfies the following integral equation:

$$p_\infty(p_0) = p_\infty^{(1)}(p_0) + \int dr K(p_0, r)\, p_\infty(r), \quad (1)$$

  where $p_\infty^{(1)}(p_0)$ and $K(p_0, r)$ are known functions. This equation can be solved iteratively, the first iteration being $p_\infty^{(1)}(p_0)$. The nth iteration provides an exact solution for $|p_0| < r_n$; for $|p_0| > r_n$ the discrepancy between the exact solution and the nth iteration can be bounded from above. Points $\{r_n\}$ form an ascending sequence with $\lim_{n\to\infty} r_n = \max\{1, m_i\} \cdot k_{\mathrm{F}}$. The asymptotic momentum is presented on Fig. 1.

- Next order corrections ($\sim g^2$) to the above results are obtained.

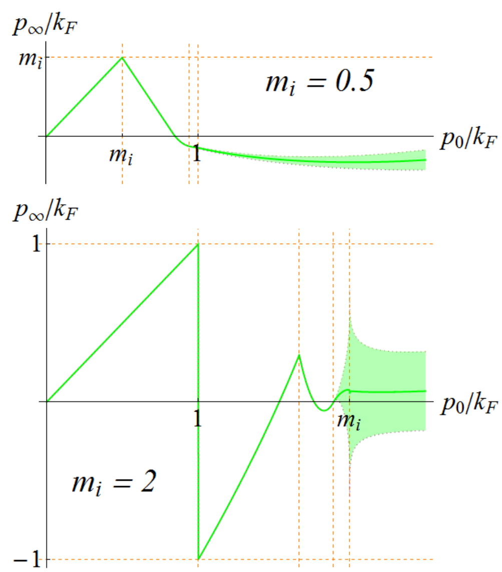

Figure 1: The asymptotic impurity momentum p_∞ as a function of the initial impurity momentum p_0 when impurity is lighter (upper plot) and heavier (lower plot) than the host particles. Solid green line – an iterative solution (two iterations) of eq. (1). This solution is exact below some point r_2 and approximate above this point. Shaded area represents the maximal error: the exact solution of eq. (1) lies inside the shaded area. Notice much better convergence of iterations in the case $m_i < 1$.

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